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New integrable problems of motion of a rigid body acted upon by nonsymmetric electromagnetic forces

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Abstract. The problem of motion of a rigid body with a spherical dynamical symmetry about a fixed point under the action of a combination of nonsymmetric nonuniform electric, magnetic, gravitational and Lorentz forces is considered. Two systems of equations of motion written in the body system and in the inertial system are shown to have the same structure. Transforming some recently found integrable cases of a nonsymmetric body in an axial combination of fields, we construct two new integrable cases of a body in a nonsymmetric combination of fields. In the latter cases, the body is assumed to have distributions of electric charges, mass and magnetization which exhibit a common axis of symmetry and carries a rotor along that axis. Explicit solution of the equations of motion is discussed.

1. Introduction

The problem of motion of a rigid body about a fixed point under the action of conservative potential and gyroscopic forces was treated in great detail only in the case when these forces admit a common axis of symmetry passing through the fixed point. In this general setting of the problem, seven general integrable cases are known at present [1]. Those cases include and generalize all the integrable cases previously known in various restricted versions of this problem, such as the three well known cases due to Euler, Lagrange and Kovalevskaya of the heavy body in a uniform field [14, 15], and the six cases of a body in a liquid [18]. In addition to general integrable cases, a large number of conditional integrable cases and particular solutions was found in several special versions of the problem [14, 15, 17, 16].

In mechanics, the knowledge of a sufficient number of integrals of motion usually guarantees integrability either in the sense of Jacobi or in the sense of Liouville. However, the procedures suggested to reduce the complete explicit solution to quadratures are rarely effective. In the majority of cases we need to look for a suitable approach to the solution of each integrable problem. This has only been achieved for a few of the known cases (see e.g. [8–13]). For an account of solved and unsolved cases see [16].

Despite its practical importance, the problem of motion under the action of nonsymmetric forces has escaped attention for a long time. Despite the richness in its structure, integrable cases of this problem are sill rare. The first one was found in [19] (see also [21]). A few more cases were introduced in [22, 3, 4, 23, 5, 6]. The case presented in [5] is the only one that involves, in addition to potential forces, not only a gyrostatic moment fixed in the body, but also gyroscopic moments depending on the orientation of the

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body. A physical interpretation of such moments is possible as a result of the Lorentz effect on a permanent distribution of charges carried by the moving body [2, 5]. An alternative explanation of those moments pointed out in [2] assumes the presence of (nonisotropic) dielectric parts of the body under the joint action of electric and magnetic fields.

In this paper we study the general problem of motion of a rigid body of complete dynamical symmetry about a fixed point under the action of a system of potential and gyroscopic forces admitting no axial symmetry. Generally speaking, this problem can be modelled by the motion of an electrified, magnetized gyrostat under the action of a skew combination of Newtonian, Coulomb, magnetic and Lorentz forces.

Our main objective is to establish certain equivalence between versions of this problem and the well-studied case of axisymmetric forces. This equivalence reveals two new integrable cases of our problem and certain connection between other cases known before. In addition, it furnishes a simple way for certain analytical and qualitative studies of the motion and usually enables complete solution of the new problem just by transforming a known solution. In both new cases, as in the case of [5], the Lorentz forces play an essential role.

We first introduce the following notation: let $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$ be the unit vectors along the axes of the inertial system $\mathrm{O}\xi\eta\zeta$ and $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ be the unit vectors along the axes of the system $\mathrm{O}xyz$, fixed in the body. The relative position of the two systems will be specified by the Eulerian angles: ψ is the angle of precession around the ζ-axis, θ is the angle of nutation between z and ζ, and φ is the angle of rotation of the body around the z-axis.

Throughout this paper we shall use the following notation. A vector $\boldsymbol{u} = (u_1, u_2, u_3)$ will always be referred to the body system, while the same vector referred to the space system will be denoted by

$$\bar{\boldsymbol{u}} = (\bar{u}_1, \bar{u}_2, \bar{u}_3) = (\boldsymbol{u} \cdot \boldsymbol{\alpha}, \boldsymbol{u} \cdot \boldsymbol{\beta}, \boldsymbol{u} \cdot \boldsymbol{\gamma}). \tag{1}$$

For example, we have

$$\bar{\boldsymbol{i}} = (\alpha_1, \beta_1, \gamma_1) \qquad \bar{\boldsymbol{j}} = (\alpha_2, \beta_2, \gamma_2) \qquad \bar{\boldsymbol{k}} = (\alpha_3, \beta_3, \gamma_3).$$

Let the body be in motion under the action of generalized conservative forces whose generalized (velocity-dependent) potential can be written in the form $V - \boldsymbol{l} \cdot \boldsymbol{\omega}$. Here $V = V(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$, $\boldsymbol{l} = \boldsymbol{l}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ are certain functions of ψ, θ, φ through the nine direction cosines $\alpha_1, \ldots, \gamma_3$ that characterize potential and gyroscopic (zero-potential) forces. This system is characterized by the Lagrangian

$$\mathcal{L} = \tfrac{1}{2} A \boldsymbol{\omega}^2 + \boldsymbol{l} \cdot \boldsymbol{\omega} - V. \tag{2}$$

As in [1], the potential V can be understood as due to certain gravitational, electric and scalar magnetic interactions. A constant term in the vector $\boldsymbol{l}$ is a gyrostatic moment, while the variable terms in $\boldsymbol{l}$ appear as a result of the Lorentz effect of the magnetic field on the electric charges. It can be expressed in the form

$$\boldsymbol{l} = \boldsymbol{m} + \int \boldsymbol{r} \times \boldsymbol{A} \, \mathrm{d}e$$

where $\boldsymbol{A}$ is the vector potential of the magnetic field at the point containing the charge element $\mathrm{d}e$ and the integral is taken over the charge distribution on the body [2]†.

We shall write the equations of motion in the vector Euler–Poisson form, which has, in many cases, certain advantages over the Lagrangian form. The equations of motion of the

† Here MKS units are used. In Gaussian units $\mathrm{d}e$ should be divided by the velocity of light c. We also assume that the velocity and acceleration are sufficiently small to neglect both relativistic effects and classical radiation damping.

general body in such a situation were derived in [7]. For a dynamically spherical body they take the form:

$$\begin{aligned} &A\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{\mu} = \boldsymbol{\alpha} \times \frac{\partial V}{\partial \boldsymbol{\alpha}} + \boldsymbol{\beta} \times \frac{\partial V}{\partial \boldsymbol{\beta}} + \boldsymbol{\gamma} \times \frac{\partial V}{\partial \boldsymbol{\gamma}} \\ &\dot{\boldsymbol{\alpha}} + \boldsymbol{\omega} \times \boldsymbol{\alpha} = \mathbf{0} \qquad \boldsymbol{\beta} + \boldsymbol{\omega} \times \boldsymbol{\beta} = \mathbf{0} \qquad \dot{\boldsymbol{\gamma}} + \boldsymbol{\omega} \times \boldsymbol{\gamma} = \mathbf{0} \end{aligned} \tag{3}$$

where A is the common moment of inertia of the body about an axis through the fixed point and

$$\boldsymbol{\mu} = \boldsymbol{l} + \left(\boldsymbol{\alpha} \times \frac{\partial}{\partial \boldsymbol{\alpha}} + \boldsymbol{\beta} \times \frac{\partial}{\partial \boldsymbol{\beta}} + \boldsymbol{\gamma} \times \frac{\partial}{\partial \boldsymbol{\gamma}} \right) \times \boldsymbol{l}. \tag{4}$$

The vector $\boldsymbol{\mu}$ can be expressed directly in terms of the gyrostatic moment and the magnetic field

$$\boldsymbol{\mu} = \boldsymbol{m} - \int (\boldsymbol{r} \cdot \boldsymbol{H}) \boldsymbol{r} \, \mathrm{d}e. \tag{5}$$

From the point of view of integrability, it may be simpler, however, to think of the mechanical system characterized by (3); or, equivalently, by (2) as a conservative Hamiltonian system of three degrees of freedom, for which only one integral is known, namely the Jacobi integral

$$I_l = \tfrac{1}{2} A \boldsymbol{\omega}^2 + V. \tag{6}$$

Complete integrability in the sense of Liouville requires the knowledge of only two additional integrals in involution. Although we shall rely on the last concept of integrability, it will be usually easier to rely on the symmetric vector form of the equations of motion (3) in constructing the two missing integrals.

2. A transformation of the equations of motion

Now we try to write down the system of equations (3) in an equivalent form referred to the inertial system $\mathrm{O}\xi\eta\zeta$. Instead of $\boldsymbol{\omega}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$ we shall use as variables $\bar{\boldsymbol{\omega}}, \bar{\boldsymbol{i}}, \bar{\boldsymbol{j}}, \bar{\boldsymbol{k}}$. Let v and $\bar{\boldsymbol{\mu}}$ denote the scalar and vector functions which now depend on $\bar{\boldsymbol{i}}, \bar{\boldsymbol{j}}, \bar{\boldsymbol{k}}$. We can use the same method of [7] or directly project the dynamical Euler equation in (3) with v and $\boldsymbol{L}$ on the directions of the fixed axes ζ, η, ζ. After some manipulations we can write the new dynamical equation as

$$A \frac{\mathrm{d}}{\mathrm{d}t} \bar{\boldsymbol{\omega}} + \bar{\boldsymbol{\omega}} \times \bar{\boldsymbol{\mu}} = -\left(\bar{\boldsymbol{i}} \times \frac{\partial v}{\partial \bar{\boldsymbol{i}}} + \bar{\boldsymbol{j}} \times \frac{\partial v}{\partial \bar{\boldsymbol{j}}} + \bar{\boldsymbol{k}} \times \frac{\partial v}{\partial \bar{\boldsymbol{k}}} \right). \tag{7}$$

Equation (7) should be augmented by the equalities that describe the space rate of change of the vectors $\bar{\boldsymbol{i}}, \bar{\boldsymbol{j}}, \bar{\boldsymbol{k}}$. In the final form the system of equations of motion in the inertial system takes the form

$$\begin{aligned} &-A \frac{\mathrm{d}}{\mathrm{d}t} \bar{\boldsymbol{\omega}} + \bar{\boldsymbol{\omega}} \times \bar{\boldsymbol{M}} = \bar{\boldsymbol{i}} \times \frac{\partial v}{\partial \bar{\boldsymbol{i}}} + \bar{\boldsymbol{j}} \times \frac{\partial v}{\partial \bar{\boldsymbol{j}}} + \bar{\boldsymbol{k}} \times \frac{\partial v}{\partial \bar{\boldsymbol{k}}} \\ &-\frac{\mathrm{d}\bar{\boldsymbol{i}}}{\mathrm{d}t} + \bar{\boldsymbol{\omega}} \times \bar{\boldsymbol{i}} = \mathbf{0} \qquad -\frac{\mathrm{d}\bar{\boldsymbol{j}}}{\mathrm{d}t} + \bar{\boldsymbol{\omega}} \times \bar{\boldsymbol{j}} = \mathbf{0} \qquad -\frac{\mathrm{d}\bar{\boldsymbol{k}}}{\mathrm{d}t} + \bar{\boldsymbol{\omega}} \times \bar{\boldsymbol{k}} = \mathbf{0} \end{aligned} \tag{8}$$

in which $\bar{\boldsymbol{M}} = -\bar{\boldsymbol{\mu}}$.

Now we easily notice that the systems (8) has the same structure as (3). They can be made fully identical by the replacement

$$\boldsymbol{\omega}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, V, \boldsymbol{\mu}(\text{or } \boldsymbol{l}), t \rightarrow \bar{\boldsymbol{\omega}}, \bar{\boldsymbol{i}}, \bar{\boldsymbol{j}}, \bar{\boldsymbol{k}}, v, \bar{\boldsymbol{M}}(\text{or } \bar{\boldsymbol{l}}), -t. \tag{9}$$

We can draw the following useful conclusion.

Let us be given an integrable case of (3) with

$$\begin{aligned} V &= v(\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3) \\ \mu_i &= \mu_i(\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3) \qquad i = 1, 2, 3 \end{aligned} \tag{10}$$

and let the general solution of the equations of motion be

$$\begin{aligned} \boldsymbol{\omega} &= \boldsymbol{\Omega}(t) \\ \boldsymbol{S} &\equiv \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} = \boldsymbol{R}(t). \end{aligned} \tag{11}$$

This case implies the integrability of the system (8) with

$$\begin{aligned} v &= V(\bar{i}_1, \bar{i}_2, \bar{i}_3, \bar{j}_1, \bar{j}_2, \bar{j}_3, \bar{k}_1, \bar{k}_2, \bar{k}_3) \\ \bar{M}_i &= -\mu_i(\bar{i}_1, \bar{i}_2, \bar{i}_3, \bar{j}_1, \bar{j}_2, \bar{j}_3, \bar{k}_1, \bar{k}_2, \bar{k}_3) \qquad i = 1, 2, 3 \end{aligned} \tag{12}$$

and also gives its general solution

$$\bar{\boldsymbol{\omega}} = \boldsymbol{\Omega}(-t) \tag{13}$$

$$\begin{pmatrix} \bar{\boldsymbol{i}} \\ \bar{\boldsymbol{j}} \\ \bar{\boldsymbol{k}} \end{pmatrix} = \boldsymbol{R}(-t). \tag{14}$$

Expressing (12) in the body system we obtain an integrable case of (3) in which $V, \boldsymbol{\mu}$ are replaced by

$$\begin{aligned} v &= V(\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2, \alpha_3, \beta_3, \gamma_3) \\ \boldsymbol{M} &= -\mu_1(\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2, \alpha_3, \beta_3, \gamma_3)\boldsymbol{\alpha} - \mu_2(\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2, \alpha_3, \beta_3, \gamma_3)\boldsymbol{\beta} \\ &\quad -\mu_3(\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2, \alpha_3, \beta_3, \gamma_3)\boldsymbol{\gamma}. \end{aligned} \tag{15}$$

Moreover, the general solution of the new case can be written as

$$\begin{aligned} \boldsymbol{\omega} &= \boldsymbol{\Omega}(-t)\boldsymbol{R}^T(-t) \\ \boldsymbol{S} &= \boldsymbol{R}^T(-t). \end{aligned} \tag{16}$$

Apart from certain special cases, the pairs of expressions (10) and (15) characterize physically different problems. The two problems become mathematically equivalent when the second is put in the form (12).

3. New integrable cases

In [1] we introduced some integrable cases of the motion of a body under the action of axisymmetric forces. Of those cases two, the second and third, are valid for a body whose ellipsoid of inertia is a sphere. We now present the equivalent of those cases in the above analogy.

3.1. The first case

For the third case of [1], the equivalent case according to section 2 is characterized by the expressions

$$
\begin{aligned}
V &= s_1\alpha_3 + s_2\beta_3 + s_3\gamma_3 - \frac{1}{2A}(bc\alpha_3^2 + ca\beta_3^2 + ab\gamma_3^2) - \tfrac{1}{2}A(n + n_1\alpha_3 + n_2\beta_3 + n_3\gamma_3)^2 \\
&\quad + \tfrac{1}{2}(n + n_1\alpha_3 + n_2\beta_3 + n_3\gamma_3)[(b+c)\alpha_3^2 + (c+a)\beta_3^2 + (a+b)\gamma_3^2] \\
\mu_1 &= -[A(n_1\alpha_1 + n_2\beta_1 + n_3\gamma_1) + a\alpha_3\alpha_1 + b\beta_3\beta_1 + c\gamma_3\gamma_1] \\
\mu_2 &= -[A(n_1\alpha_2 + n_2\beta_2 + n_3\gamma_2) + a\alpha_3\alpha_2 + b\beta_3\beta_2 + c\gamma_3\gamma_2] \\
\mu_3 &= A(n + n_1\alpha_3 + n_2\beta_3 + n_3\gamma_3) - (a\alpha_3^2 + b\beta_3^2 + c\gamma_3^2).
\end{aligned}
\tag{17}
$$

For the sake of clarity and for direct verification of the results we write down the integrals of the new integrable problem (17). They have the form:

$$
I_2 = Ar - \tfrac{1}{2}[(b+c)\alpha_3^2 + (c+a)\beta_3^2 + (a+b)\gamma_3^2] + A(n + n_1\alpha_3 + n_2\beta_3 + n_3\gamma_3) \tag{18}
$$

$$
\begin{aligned}
I_3 &= \tfrac{1}{2}A\{(b+c)[\boldsymbol{\omega}\cdot\boldsymbol{\alpha} + (n + n_1\alpha_3 + n_2\beta_3 + n_3\gamma_3)\alpha_3]^2 \\
&\quad + (c+a)[\boldsymbol{\omega}\cdot\boldsymbol{\beta} + (n + n_1\alpha_3 + n_2\beta_3 + n_3\gamma_3)\beta_3]^2 \\
&\quad + (a+b)[\boldsymbol{\omega}\cdot\boldsymbol{\gamma} + (n + n_1\alpha_3 + n_2\beta_3 + n_3\gamma_3)\gamma_3]^2\} \\
&\quad + (s_1 - n_1I_2)[A(\boldsymbol{\omega}\cdot\boldsymbol{\alpha} + (n + n_1\alpha_3 + n_2\beta_3 + \gamma_3n_3)\alpha_3) + a\alpha_3] \\
&\quad + (s_2 - n_2I_2)[A(\boldsymbol{\omega}\cdot\boldsymbol{\beta} + (n + n_1\alpha_3 + n_2\beta_3 + \gamma_3n_3)\beta_3) + b\beta_3] \\
&\quad + (s_3 - n_3I_2)[A(\boldsymbol{\omega}\cdot\boldsymbol{\gamma} + (n + n_1\alpha_3 + n_2\beta_3 + \gamma_3n_3)\gamma_3) + c\gamma_3] \\
&\quad - abc\left\{[\boldsymbol{\omega}\cdot\boldsymbol{\alpha} + (n + n_1\alpha_3 + n_2\beta_3 + \gamma_3n_3)\alpha_3]\frac{\alpha_3}{a}\right. \\
&\quad + [\boldsymbol{\omega}\cdot\boldsymbol{\beta} + (n + n_1\alpha_3 + n_2\beta_3 + \gamma_3n_3)\beta_3]\frac{\beta_3}{b} \\
&\quad \left. + [\boldsymbol{\omega}\cdot\boldsymbol{\gamma} + (n + n_1\alpha_3 + n_2\beta_3 + \gamma_3n_3)\gamma_3]\frac{\gamma_3}{c}\right\}.
\end{aligned}
\tag{19}
$$

The integral I_2 is linear in velocities. It indicates that the angle of proper rotation φ is a cyclic variable. The moving body should be symmetric around its z-axis. The integral I_3 is a polynomial of the second degree in velocities with coefficients depending on $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$.

3.1.1. The explicit solution To obtain an explicit solution of the case (17), it suffices to apply the transformation (16) to the solution (11) of the original case. The latter is known only in a very special case of motion of a body of liquid, due to Lyapunov [20], and characterized by $s_1 = s_2 = s_3 = n = n_1 = n_2 = n_3 = 0$. This case was solved by Kötter in terms of theta functions of two arguments [9]. According to (16) the solution of (17) for $s_1 = s_2 = s_3 = n = n_1 = n_2 = n_3 = 0$ can be expressed in the same class of functions.

3.2. The second case

This is the equivalent of the second case of [1], and it is characterized by

$$
\begin{aligned}
V &= \tfrac{1}{2}(c_1\alpha_3^2 + c_2\beta_3^2 + c_3\gamma_3^2) - \tfrac{1}{2}A(n + n_1\alpha_3^2 + n_2\beta_3^2 + n_3\gamma_3^2)^2 \\
\mu_1 &= -2A(n_1\alpha_3\alpha_1 + n_2\beta_3\beta_1 + n_3\gamma_3\gamma_1) \\
\mu_2 &= -2A(n_1\alpha_3\alpha_2 + n_2\beta_3\beta_2 + n_3\gamma_3\gamma_2) \\
\mu_3 &= A(n + n_1\alpha_3^2 + n_2\beta_3^2 + n_3\gamma_3^2).
\end{aligned}
\tag{20}
$$

This problem admits the integrals:

$$I_2 = A(r + n + n_1\alpha_3^2 + n_2\beta_3^2 + n_3\gamma_3^2) \tag{21}$$

$$\begin{aligned} I_3 = A((c_1 - 2n_1 I_2)(\boldsymbol{\omega}\cdot\boldsymbol{\alpha} + (n + n_1\alpha_3^2 + n_2\beta_3^2 + n_3\gamma_3^2)\alpha_3)^2 \\ +(c_2 - 2n_2 I_2)(\boldsymbol{\omega}\cdot\boldsymbol{\beta} + (n + n_1\alpha_3^2 + n_2\beta_3^2 + n_3\gamma_3^2)\beta_3)^2 \\ +(c_3 - 2n_3 I_2)(\boldsymbol{\omega}\cdot\boldsymbol{\gamma} + (n + n_1\alpha_3^2 + n_2\beta_3^2 + n_3\gamma_3^2)\gamma_3)^2) \\ -(2n_2 I_2 - c_2)(2n_3 I_2 - c_3)\alpha_3^2 - (2n_3 I_2 - c_3)(2n_1 I_2 - c_1)\beta_3^2 \\ -(2n_1 I_2 - c_1)(2n_2 I_2 - c_2)\gamma_3^2. \end{aligned} \tag{22}$$

Note that in (22) I_2 stands for its expression (21). In the general case, the integral I_3 is a polynomial of the third degree in the velocities with coefficients depending on $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$. In the special case, when $n_1 : n_2 :: n_3 :: c_1 : c_2 : c_3$, a constant factor can be omitted and the integral becomes of the second degree. The same happens when $n_1 = n_2 = n_3 = 0$.

3.2.1. The explicit solution The original case generalizes by the introduction of the three parameters n_1, n_2, n_3 the case of motion of a body in liquid known as Clebsch's case of complete dynamical symmetry and reduces to that case when $n = n_1 = n_2 = n_3 = 0$ [1]. The solution of Clebsch's case can be expressed in terms of theta functions of two variables [10] and so will be the solution of the second new case (20) through formulae (16).

4. Conclusion

In this paper we have studied the problem of motion about a fixed point of a rigid body gyrostat with a spherical inertia tensor under the action of an asymmetric combination of gravitational, electric, magnetic and Lorentz forces. The main results can be summarized as follows.

(1) We show that the two systems of equations of motion written in the body system and in the inertial system have the same structure and are connected by a simple transformation.

(2) We use this situation to construct new integrable problems from different ones that are known to be integrable. In particular, we generate two new integrable cases of motion in which the body is axisymmetric and moves in a nonsymmetric combination of fields from two known cases in which the body is not axisymmetric but moves in an axisymmetric combination of fields.

(3) We show that certain versions of the new cases can be explicitly solved in terms of theta functions of two variables by transforming known solutions due to Kötter in a classical problem of motion of a body in a liquid.

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